Exercise 19

Guess the value of the limit (if it exists) by evaluating the function at the given numbers (correct to six decimal places).

$$\lim_{x\to 3}\frac{x^2-3x}{x^2-9},$$

$$x = 3.1, 3.05, 3.01, 3.001, 3.0001,$$
$$2.9, 2.95, 2.99, 2.999, 2.9999$$

Solution

Evaluate the given function at all the given values of x.

$$\left.\frac{x^2-3x}{x^2-9}\right|_{x=2.9} = 0.491525$$

$$\left.\frac{x^2-3x}{x^2-9}\right|_{x=2.95} = 0.495798$$

$$\left.\frac{x^2-3x}{x^2-9}\right|_{x=2.99} = 0.499165$$

$$\left.\frac{x^2-3x}{x^2-9}\right|_{x=2.999} = 0.499917$$

$$\left.\frac{x^2-3x}{x^2-9}\right|_{x=2.9999} = 0.499992$$

$$\left.\frac{x^2-3x}{x^2-9}\right|_{x=3.0001} = 0.500008$$

$$\left.\frac{x^2-3x}{x^2-9}\right|_{x=3.001} = 0.500083$$

$$\left.\frac{x^2-3x}{x^2-9}\right|_{x=3.01} = 0.500832$$

$$\left.\frac{x^2-3x}{x^2-9}\right|_{x=3.05} = 0.504132$$

$$\left.\frac{x^2-3x}{x^2-9}\right|_{x=3.1} = 0.508197$$

Therefore,

$$\lim_{x\to 3}\frac{x^2-3x}{x^2-9} = 0.5 = \frac{1}{2}.$$